DAY NINE

Gravitation

Learning & Revision for the Day

- Universal Law of Gravitation
- Acceleration due to Gravity
- Gravitational Field
- Gravitational Potential
- Gravitational Potential Energy
- Escape Velocity
- Artificial Satellite
- Geostationary Satellite
- Kepler's Laws of Planetary Motion

Universal Law of Gravitation

In this universe, each body attracts other body with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Let m_1 and m_2 be the masses of two bodies and r be the separation between them.

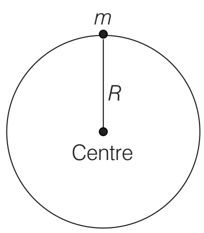
$$F = G\frac{m_1 m_2}{r^2}.$$

The proportionality constant G is called **universal gravitational constant**. In SI system, value of gravitational constant G is $6.67 \times 10^{-11}\,\text{Nm}^2\text{kg}^{-2}$. Dimensional formula of G is $[\text{M}^{-1}\text{L}^3\text{T}^{-2}]$.

Acceleration due to Gravity

The acceleration of an object during its free fall towards the earth is called acceleration due to gravity.

If M is the mass of earth and R is the radius, the earth attracts a mass m on its surface with a force F given by



$$F = \frac{GMm}{R^2}$$

This force imparts an acceleration to the mass m which is known as acceleration due to gravity (g).

By Newton's law, we have

$$\text{Acceleration } (g) = \frac{F}{m} = \frac{\frac{GMm}{R^2}}{m} = \frac{GM}{R^2}$$

On the surface of earth, $g = \frac{GM}{R^2}$

Substituting the values of G, M, R, we get $g = 9.81\ \text{ms}^{-2}$.

Mass of the earth $m = 6 \times 10^{24}$ kg and radius of the earth $R = 6.4 \times 10^6$ m.

Variation in g with Altitude and Depth

The value of g is variable and can vary in same cases as mentioned below

- **Value of acceleration due to gravity (g) at a height (h)** from the surface of the earth is given by

$$g' = \frac{gR^2}{(R+h)^2}$$

If $h << R$, then $g' = g\left[1 - \frac{2h}{R}\right]$

- **Value of acceleration due to gravity (g) at a depth (d)** from the surface of the earth is given by

$$g' = g\left(1 - \frac{d}{R}\right)$$

At the centre of the earth $d = R$ and hence, $g' = 0$.

Variation in the Value of (g) Due to Rotation of the Earth

Due to rotation of the earth, the value of g decreases as the speed of rotation of the earth increases. The value of acceleration due to gravity at a latitude is $g'_\lambda = g - R\omega^2 \cos^2 \lambda$

Following conclusions can be drawn from the above discussion

- The effect of centrifugal force due to rotation of the earth is to reduce the effective value of g.
- The effective value of g is not truely in vertical direction.
  At the equators, $\lambda = 0°$
  Therefore, $g' = g - R\omega^2$ (minimum value)
- At the poles, $\lambda = 90°$
  Therefore, $g' = g$ (maximum value)

Gravitational Field

The space surrounding a material body in which its gravitational force of attraction can be experienced is called its gravitational field.

Gravitational Field Intensity

Gravitational field intensity at any point is defined as the force experienced by any test mass devided by the magnitude of test mass when placed at the desired point.

Mathematically,

Gravitational field intensity, $\mathbf{E} = \frac{\mathbf{F}}{m_0}$

where, m_0 is a small test mass. The SI unit of gravitational intensity is N kg^{-1}.

- Gravitational intensity at a point situated at a distance r from a point mass M is given by

$$\mathbf{E} = \frac{GM}{r^2}$$

- Gravitational field intensity due to a solid sphere (e.g. earth) of mass M and radius R at a point distant r from its centre $(r > R)$ is $\mathbf{E} = \frac{GM}{r^2}$

  and at the surface of solid sphere, $\mathbf{E} = \frac{GM}{R^2}$.

  However, for a point $r < R$, we find that

$$\mathbf{E} = \frac{GMr}{R^3}$$

- Due to a body in the form of uniform shell gravitational field intensity at a point outside the shell $(r > R)$ is given by $\mathbf{E} = \frac{GM}{r^2}$

  But at any point inside the shell, gravitational intensity is zero.

Gravitational Potential

Gravitational potential at any point in a gravitational field is defined as the work done in bringing a unit mass from infinity to that point.

Gravitational potential, $V = \lim_{m_0 \to 0} \frac{W}{m_0}$.

Gravitational potential due to a point mass is $V = -\frac{GM}{r}$.

Gravitational potential is always negative. It is a scalar term and its SI unit is J kg^{-1}.

For Solid Sphere

- At a point outside the solid sphere, (e.g. earth), i.e.

$$r > R, \ V = -\frac{GM}{r}.$$

- At a point on the surface of spherical,

$$V = -\frac{GM}{R}.$$

- At a point inside the sphere, $(r < R)$.

$$V = -\frac{GM}{2R^3}(3R^2 - r^2) = -\frac{GM}{2R}\left[3 - \frac{r^2}{R^2}\right]$$

- At the centre of solid sphere,

$$V = -\frac{3GM}{2R} = \frac{3}{2}V_{\text{surface}}$$

For Spherical Shell

- At a point outside the shell,

$$V = -\frac{GM}{r} \text{ where, } r > R.$$

- At a point on the surface of spherical shell,

$$V = -\frac{GM}{R}$$

- At any point inside the surface of spherical shell

$$V = -\frac{GM}{R} = V_{\text{surface}}$$

Relation between Gravitational Field and Gravitational Potential

If $\mathbf{r}_1$ and $\mathbf{r}_2$ are position of two points in the gravitation field ($\mathbf{I}$), then change in gravitational potential

$$V(\mathbf{r}_2) - V(\mathbf{r}_1) = -\int_{r_1}^{r_2} \mathbf{I} \cdot d\mathbf{r}$$

$\Rightarrow$ $dV = -\mathbf{I} \cdot d\mathbf{r} = I_x \hat{\mathbf{i}} + I_y \hat{\mathbf{j}} + I_z \hat{\mathbf{k}}$, then

and $dr = dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}} + dz\,\hat{\mathbf{k}}$

$$= (I_x\,\hat{\mathbf{i}} + I_y\,\hat{\mathbf{j}} + I_z\hat{\mathbf{k}}),$$

$$dV = -\mathbf{I}.d\mathbf{r} = -I_x dx - I_y dy - I_z\,dz$$

Thus, $\mathbf{I} = -\frac{\partial V}{\partial x}\hat{\mathbf{i}} - \frac{\partial V}{\partial y}\hat{\mathbf{j}} - \frac{\partial V}{\partial z}\hat{\mathbf{k}}$

Remember that partial differentiation indicates that variation of gravitational potential in counter along the variation of x-coordinate, then other coordinates (i.e. y and z) are assumed to be constant.

Gravitational Potential Energy

Gravitational potential energy of a body or system is negative of work done by the conservative gravitational forces F in bringing it from infinity to the present position. Mathematically, gravitational potential energy

$$U = -W = -\int_{\infty}^{\mathbf{r}} \mathbf{F} \cdot d\mathbf{r}$$

- The gravitational potential energy of two particles of masses m_1 and m_2 separated by a distance r is given by

$$U = -\frac{Gm_1m_2}{r}$$

- The gravitational potential energy of mass m at the surface of the earth is

$$U = -\frac{GMm}{R}$$

- Difference in potential energy of mass m at a height h from the earth's surface and at the earth's surface is

$$U_{(R+h)} - U_R = \frac{mgh}{1 + \frac{h}{R}}$$

$$\simeq mgh, \text{ if } h << R$$

- For three particles system,

$$U = -\left[\frac{Gm_1m_2}{r_{12}} + \frac{Gm_1m_3}{r_{13}} + \frac{Gm_2m_3}{r_{23}}\right]$$

- For n-particles system, $\frac{n(n-1)}{2}$ pairs form and total potential energy of the system is sum of potential energies of all such pairs.

Escape Velocity

It is the minimum velocity with which a body must be projected from the surface of the earth so that it escapes the gravitational field of the earth. We can also say that a body, projected with escape velocity, will be able to go to a point which is at infinite distance from the earth.

The value of escape velocity from the surface of a planet of mass M, radius R and acceleration due to gravity g is

$$v_{\text{escape}} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

$$v_{\text{escape}} = \sqrt{2}v_{\text{orbital}}$$

Escape velocity does not depend upon the mass or shape or size of the body as well as the direction of projection of the body. For earth value of escape velocity is 11.2 kms^{-1}.

Artificial Satellites

Artificial satellites are man made satellites launched from the earth. The path of these satellites are elliptical with the centre of earth at a foci of the ellipse. However, as a first approximation we may consider the orbit of satellite as circular.

Orbital Velocity of Satellite

Orbital velocity of a satellite is the velocity required to put the satellite into its orbit around the earth. The orbital velocity of satellite is given by

$$v_o = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR^2}{r}} = \sqrt{\frac{gR^2}{(R+h)}}$$

If $h << R$ or $r \simeq R$, then

$$v_o = \sqrt{\frac{GM}{R}} = \sqrt{gR} = 7.9\ \text{kms}^{-1}$$

Period of Revolution

It is the time taken by a satellite to complete one revolution around the earth.

Revolution period, $T = \frac{2\pi r}{v_o} = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{r^3}{gR^2}} = \sqrt{\frac{3\pi}{G.e}}$

If $r \simeq R$, then $T = 2\pi\sqrt{\frac{R}{g}} = 84.6$ min.

Height of Satellite in Terms of Period

The height of the satellite (from the earth planet) can be determined by its time period and *vice-versa*.

As the height of the satellite in terms of time period,

$$h = r - R = \left[\frac{gR^2T^2}{4\pi^2}\right]^{1/3} - R.$$

Energy of Satellite

Kinetic energy of satellite, $K = \frac{1}{2} m v_0^2 = \frac{GMm}{2r}$.

Potential energy of satellite, $U = -\frac{GMm}{r}$

and total energy of satellite $E = K + U = -\frac{GMm}{2r} = -K$.

Binding Energy of Satellite

It is the energy required to remove the satellite from its orbit and take it to infinity.

Binding energy $= -E = +\frac{GMm}{2r}$

Angular Momentum of Satellite

Angular momentum of a satellite, $L = m v_0 r = \sqrt{m^2 GMr}$

Geostationary Satellite

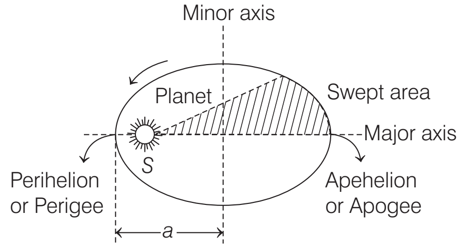
If an artificial satellite revolves around the earth in an equatorial plane with a time period of 24 h in the same sense as that of the earth, then it will appear stationary to the observer on the earth. Such a satellite is known as a **geostationary satellite** or **parking satellite**.

Kepler's Laws of Planetary Motion

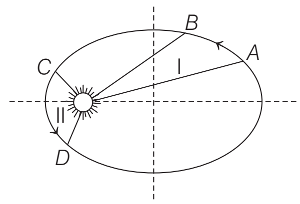
Kepler discovered three empirical laws which accurately describe the motion of planets.

These laws are

1. **Law of Orbits** All the planets move around the sun in an elliptical orbit with sun at one of the focus of ellipse.



2. **Law of Areas** The line joining the sun to the planet sweeps out equal areas in equal intervals of time, i.e. areal velocity of the planet w.r.t. sun is constant. This is called the law of area, which indicates that a planet moves faster near the sun and slowly when away from the sun.



3. **Law of Periods** The square of the planet's time period of revolution is directly proportional to the cube of semi-major axis of its orbit.

$$T^2 \propto a^3$$

where a is the semi-major axis.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 The magnitude of the force of gravity between two identical objects is given by F_0. If the mass of each object is doubled but the distance between them is halved, then the new force of gravity between the object will be

(a) $16F_0$ (b) $4F_0$ (c) F_0 (d) $F_0/2$

2 Two astronauts are floating in gravitational free space after having lost contact with their spaceship. The two will → NEET 2017

(a) keep floating at the same distance between them
(b) move towards each other
(c) move away from each other
(d) will become stationary

3 Two spherical bodies of masses M and $5M$ and radii R and $2R$ are released in free space with initial separation between their centres equal to $12R$. If they attract each other due to gravitational force only, then the distance covered by the smaller body before collision is → CBSE AIPMT 2015

(a) 2.5 R (b) 4.5 R (c) 7.5 R (d) 1.5 R

4 Force between two objects of equal masses is F. If 25% mass of one object is transferred to the other object, then the new force will be

(a) $\frac{F}{4}$ (b) $\frac{3F}{4}$ (c) $\frac{15F}{16}$ (d) F

5 Let F_1 is the gravitational force experienced by a particle at a height $\frac{\sqrt{3}-1}{2\sqrt{2}} R_e$ above the earth's surface and F_2 is the gravity force experienced by a particle at a depth $\frac{\sqrt{3}-1}{2\sqrt{2}} R_e$ below the earth's surface, then (R_e is radius of earth)

(a) $F_1 = F_2$ (b) $F_1 > F_2$
(c) $F_1 < F_2$ (d) None of these

6 The depth d at which the value of acceleration due to gravity becomes $\frac{1}{n}$ times the value at the surface,

(R = radius of the earth)

(a) $\frac{R}{n}$ (b) $R\left(\frac{n-1}{n}\right)$ (c) $\frac{R}{n^2}$ (d) $R\left(\frac{n}{n+1}\right)$

7 If the mass of the Sun were ten times smaller and the universal gravitational constant were ten times larger in magnitude, which of the following is not correct?

(a) Time period of a simple pendulum on the Earth would decrease
(b) Walking on the ground would become more difficult
(c) Raindrops will fall faster
(d) 'g' on the Earth will not change

8 If the value of g at the surface of the earth is 9.8 m/s^2, then the value of g at a place 480 km above the surface of the earth will be (radius of the earth is 6400 km)

(a) 8.4 m/s^2 (b) 9.8 m/s^2 (c) 7.2 m/s^2 (d) 4.2 m/s^2

9 A what depth below the surface of the earth, the value of g is the same as that at a height of 5 km?

(a) 5 km (b) 2.5 km (c) 10 km (d) 6 km

10 Assuming the earth to be a sphere of uniform mass density, how much would body weight half way down to the centre of earth, if it weighed 250 N on the surface?

(a) 150 N (b) 175 N (c) 125 N (d) 200 N

11 The acceleration due to gravity at a height 1 km above the earth is the same as at a depth d below the surface of earth. Then ➔ NEET 2017

(a) $d = \frac{1}{2}$ km (b) d = 1km (c) $d = \frac{3}{2}$ km (d) d = 2 km

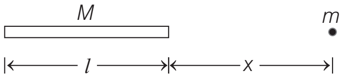
12 The height at which the weight of a body becomes 1/16th, its weight on the surface of the earth (radius R), is ➔ CBSE AIPMT 2012

(a) 5 R (b) 15 R (c) 3 R (d) 4 R

13 A spherical planet has a mass M_p and diameter D_p. A particle of mass m falling freely near the surface of this planet will experience an acceleration due to gravity, equal to ➔ CBSE AIPMT 2012

(a) $4GM_p/D_p^2$ (b) GM_pm/D_p^2 (c) GM_p/D_p^2 (d) $4GM_pm/D_p^2$

14 A mass m is at a distance x from one end of a uniform rod of length l and mass M. The gravitational force on the mass due to the rod is

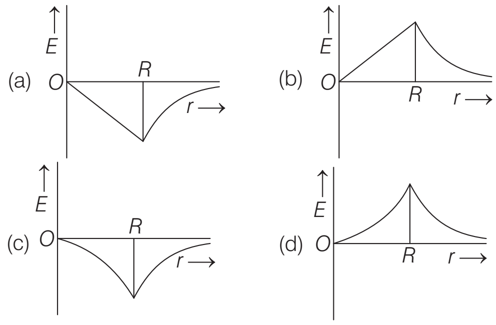


(a) $\frac{GMm}{(x+l)^2}$ (b) $\frac{GMm}{x(l+x)}$ (c) $\frac{GMm}{x^2}$ (d) $\frac{GmM}{\left(\frac{l}{2}+x\right)^2}$

15 In a gravitational field, if a body is bound with the earth, then total mechanical energy has

(a) positive (b) zero (c) negative
(d) may be positive, negative or zero

16 Dependence of intensity of gravitational field (E) of earth with distance (r) from centre of earth is correctly represented by ➔ CBSE-AIPMT 2014



17 A thin rod of length L is bent to form a semi-circle. The mass of rod is M. What will be the gravitational potential at the centre of the circle?

(a) $-\frac{GM}{L}$ (b) $-\frac{GM}{2\pi L}$ (c) $-\frac{\pi GM}{2L}$ (d) $-\frac{\pi GM}{L}$

18 Infinite number of bodies, each of mass 2 kg are situated on x-axis at distance 1 m, 2 m, 4 m, 8 m respectively from the origin. The resulting gravitational potential due to this system at the origin will be ➔ NEET 2013

(a) $-G$ (b) $-\frac{8}{3}G$ (c) $-\frac{4}{3}G$ (d) $-4G$

19 At what height from the surface of earth the gravitation potential and the value of g are -5.4×10^7 J kg^{-2} and 6.0 ms^{-2} respectively? Take, the radius of earth as 6400 km. ➔ NEET 2016

(a) 1600 km (b) 1400 km (c) 2000 km (d) 2600 km

20 A particle of mass M is situated at the centre of a spherical shell of same mass and radius a. The gravitational potential at a point situated at $a/2$ distance from the centre, will be ➔ CBSE AIPMT 2010

(a) $-\frac{3GM}{a}$ (b) $-\frac{2GM}{a}$ (c) $-\frac{GM}{a}$ (d) $-\frac{4GM}{a}$

21 A body of mass m taken from the earth's surface to the height equal to twice the radius (R) of the earth. The change in potential energy of body will be ➔ NEET 2013

(a) $mg\,2R$ (b) $\frac{2}{3}mgR$ (c) $3mgR$ (d) $\frac{1}{3}mgR$

22 When a body is lifted from surface of the earth to a height equal to the radius of the earth, then the change in its potential energy is

(a) mgR (b) $2mgR$ (c) $\frac{1}{2}mgR$ (d) 4 mgR

23 The orbital velocity of an artificial satellite in a circular orbit just above the earth's surface is v. For a satellite orbiting at an altitude of half the earth's radius the orbital velocity is

(a) $\frac{3}{2}v$ (b) $\sqrt{\frac{3}{2}}v$ (c) $\sqrt{\frac{2}{3}}v$ (d) $\frac{2}{3}v$

24 Two satellites of earth, S_1 and S_2 are moving in the same orbit. The mass of S_1 is four times the mass of S_2. Which one of the following statements is true?

(a) The time period of S_1 is four times that of S_2
(b) The potential energies of earth and satellite in the two cases are equal
(c) S_1 and S_2 are moving with the same speed
(d) The kinetic energies of the two satellites are equal

25 A remote sensing satellite of earth revolves in a circular orbit at a height of 0.25×10^6 m above the surface of earth. If earth's radius is 6.38×10^6 m and $g = 9.8\ \text{ms}^{-2}$, then the orbital speed of the satellite is ➜ CBSE AIPMT 2015

(a) $7.76\ \text{kms}^{-1}$ (b) $8.56\ \text{kms}^{-1}$ (c) $9.13\ \text{kms}^{-1}$ (d) $6.67\ \text{kms}^{-1}$

26 The radii of circular orbits of two satellites A and B of the earth are $4R$ and R, respectively. If the speed of satellite A is $3v$, then the speed of satellite B will be ➜ CBSE AIPMT 2010

(a) $3v/4$ (b) $6v$ (c) $12v$ (d) $3v/2$

27 The minimum energy required to send a 3000 kg spacecraft from the earth to a far away location where earth's gravity is negligible, would be $(M_e = 6\times 10^{24}\ \text{kg}$, $R_e = 6400\ \text{km})$

(a) 1.88×10^{11} J (b) 9×10^{10} J (c) 1×10^{11} J (d) 6×10^{11} J

28 For a satellite moving in an orbit around the earth, the ratio of kinetic energy to potential energy is

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

29 Network done on a satellite revolving in an elliptical orbit in one complete revolution

(a) is zero (b) is non-zero
(c) may be zero (d) None of these

30 An artificial satellite moving in a circular orbit around the earth has a total (kinetic + potential) energy E_0. Its potential energy is

(a) $-E_0$ (b) $1.5\ E_0$ (c) $2E_0$ (d) E_0

31 A satellite of mass m is orbiting the earth (of radius R) at a height h from its surface. The total energy of the satellite in terms of g_0, the value of acceleration due to gravity at the earth's surface is ➜ NEET 2016

(a) $\frac{mg_0R^2}{2(R+h)}$ (b) $-\frac{mg_0R^2}{2(R+h)}$ (c) $\frac{2mg_0R^2}{R+h}$ (d) $-\frac{2mg_0R^2}{R+h}$

32 An earth satellite is moved from one stable circular orbit to another higher stable circular orbit. Which one of the following quantities increase for the satellite as a result of this change?

(a) Angular momentum (b) Kinetic energy
(c) Angular velocity (d) Linear orbital speed

33 A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small as compared to the mass of the earth. Then, ➜ CBSE AIPMT 2015

(a) the angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant
(b) the total mechanical energy of S varies periodically with time
(c) the linear momentum of S remains constant in magnitude
(d) the acceleration of S is always directed towards the centre of the earth

34 A planet moving along an elliptical orbit is closest to the sun at a distance r_1 and farthest away at a distance of r_2. If v_1 and v_2 are the linear velocities at these points respectively, then the ratio $\frac{v_1}{v_2}$ is ➜ CBSE AIPMT 2011

(a) r_2/r_1 (b) $(r_2/r_1)^2$ (c) r_1/r_2 (d) $(r_1/r_2)^2$

35 The condition for a uniform spherical mass m of radius r to be a black hole is
(G = gravitational constant and g = acceleration due to gravity)

(a) $\left(\frac{2Gm}{r}\right)^{1/2} \le C$ (b) $\left(\frac{2gm}{r}\right)^{1/2} = C$
(c) $\left(\frac{2Gm}{r}\right)^{1/2} \ge C$ (d) $\left(\frac{gm}{r}\right)^{1/2} \ge C$

36 The velocity with which a projectile must be fired to escape from the earth does not depend upon

(a) mass of earth (b) mass of projectile
(c) radius of earth (d) None of these

37 A body is dropped from a height R_e (radius of earth) above the earth surface. It strikes the earth with speed v_o, if v_e is the escape velocity from earth's surface, then $\frac{v_o}{v_e}$ is

(a) $\sqrt{2}:1$ (b) $1:2$ (c) $1:\sqrt{2}$ (d) $2:1$

38 Escape velocity on the earth is $11.2\ \text{kms}^{-1}$. What would be the escape velocity on a planet whose mass is 1000 times and radius is 10 times that of earth?

(a) $112\ \text{kms}^{-1}$ (b) $11.2\ \text{kms}^{-1}$ (c) $1.12\ \text{kms}^{-1}$ (d) $3.7\ \text{kms}^{-1}$

39 A black hole is an object whose gravitational field is so strong that even light cannot escape from it. To what approximate radius would earth (mass $= 5.98\times 10^{24}$ kg) have to be compressed to a black hole? ➜ CBSE AIPMT 2014

(a) 10^{-9} m (b) 10^{-6} m (c) 10^{-2} m (d) 100 m

40 The ratio of escape velocity at earth (v_e) to the escape velocity at a planet (v_p) whose radius and mean density are twice as that of earth is ➜ NEET 2016

(a) $1:2\sqrt{2}$ (b) $1:4$ (c) $1:\sqrt{2}$ (d) $1:2$

41 If the distance between the earth and sun were half its present value, the number of days in a year would have been

(a) 64.5 (b) 129 (c) 182.5 (d) 730

42 The ratio of mean distances of three planets from the sun are $0.5:1:1.5$, then the square of time periods are in the ratio of

(a) $1:4:9$ (b) $1:9:4$ (c) $1:8:27$ (d) $2:1:3$

43 Kepler's third law states that square of period of revolution (T) of a planet around the sun, is proportional to third power of average distance r between sun and planet

i.e. $T^2 = Kr^3$.

Here, K is constant.

If the masses of sun and planet are M and m respectively, then as per Newton's law of gravitation force of attraction between them is

$F = \frac{GMm}{r^2}$, here G is gravitational constant.

The relation between G and K is described as

➔ CBSE AIPMT 2015

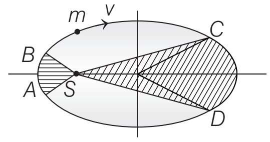
(a) $GK = 4\pi^2$ (b) $GMK = 4\pi^2$
(c) $K = G$ (d) $K = \frac{1}{G}$

44 A geostationary satellite is orbiting the earth at a height of $5R$ above that surface of the earth, R being the radius of the earth. The time period of another satellite in hour at a height of $2R$ from the surface of the earth is

➔ CBSE AIPMT 2012

(a) 5 (b) 10 (c) $6\sqrt{2}$ (d) $6/\sqrt{2}$

45 The figure shows elliptical orbit of a planet m about the sun S. The shaded area SCD is twice the shaded area SAB. If t_1 is the time for the planet to move from C to D and t_2 is the time to move from A to B, then



➔ CBSE AIPMT 2009

(a) $t_1 > t_2$ (b) $t_1 = 4t_2$ (c) $t_1 = 2t_2$ (d) $t_1 = t_2$

46 How long will a satellite, placed in a circular orbit of radius that is $\left(\frac{1}{4}\right)^{th}$ the radius of a geostationary satellite, take to complete one revolution around the earth

(a) 12 h (b) 6 h (c) 3 h (d) 4 h

47 The planet neptune travels around the sun with a period of 165 yr. What is the radius of orbit approximately, if the orbit is considered as circular?

(a) $20R_1$ (b) $30R_1$ (c) $25R_1$ (d) $35R_1$

48 At its aphelion, the planet mercury is 6.99×10^{10} m from the sun and at its perihelion it is 4.6×10^{10} m from the sun. If its orbital speed of aphelion is 3.88×10^4 ms^{-1}, then its perihelion orbital speed would be

(a) 3.88×10^4 ms^{-1} (b) 5.90×10^4 ms^{-1}
(c) 5.00×10^4 ms^{-1} (d) 5.5×10^4 ms^{-1}

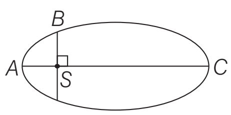
49 The time period of moon's revolution is 27.3 days and radius of the earth is 6.37×10^6 m, distance to the moon is 3.84×10^8 m, then the mass of the earth is (approximately)

(a) 10^{24} kg (b) 10^{16} kg (c) 10^{16} kg (d) 10^5 kg

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 The kinetic energies of a planet in an elliptical orbit about the Sun, at positions A, B and C are K_A, K_B and K_C, respectively. AC is the major axis and SB is perpendicular to AC at the position of the Sun S as shown in the figure. Then



(a) $K_B < K_A < K_C$ (b) $K_A > K_B > K_C$
(c) $K_A < K_B < K_C$ (d) $K_B > K_A > K_C$

2 Imagine a new planet having the same density as that of earth but it is 3 times bigger than the earth in size. If the acceleration due to gravity on the surface of earth is g and that on the surface of the new planet is g', then

(a) $g' = 3g$ (b) $g' = \frac{g}{9}$
(c) $g' = 9g$ (d) $g' = 27g$

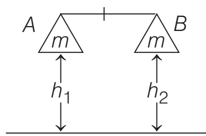
3 The distance of the centres of moon and earth is D. The mass of the earth is 81 times the mass of the moon. At what distance from the centre of the earth, the gravitational force on a particle will be zero?

(a) $\frac{D}{2}$ (b) $\frac{2D}{3}$ (c) $\frac{4D}{3}$ (d) $\frac{9D}{10}$

4 A geostationary satellite orbits around the earth in a circular orbit of radius 36000 km. Then, the time period of a spy-satellite orbiting a few 100 km above the earth's surface $(R = 6400$ km$)$ will approximately be

(a) $\frac{1}{2}$ h (b) 1 h (c) 2 h (d) 4 h

5 In the figure given below, equal mass m are hung in a balance.



The error in weighing, if $h_1 > h_2$ and R is the radius of the earth is

(a) $\frac{2mg}{R}(h_1 - h_2)$ (b) $\frac{2mg}{R}(h_2 - h_1)$
(c) $mg\left(1 - \frac{2h}{R_1}\right)$ (d) $2mg\left(\frac{h_2 + h_1}{h_1 h_2}\right)$

6 There is a crater of depth $\frac{R}{100}$ on the surface of the moon of radius R. A projectile is fired vertically upwards from the crater with a velocity equal to the escape velocity v from the surface of the moon. Maximum height attained by the projectile is
(a) $2R$ (b) $99.5R$ (c) $\frac{R}{2}$ (d) $\frac{R}{99.5}$

7 An object of mass m is raised from the surface of the earth to a height equal to the radius of the earth, that is, taken from a distance R to $2R$ from the centre of the earth. What is the gain in its potential energy?
(a) $\frac{1}{2}mgR$ (b) $\frac{1}{4}mgR$ (c) $\frac{1}{2}mgR^2$ (d) $\frac{1}{6}mgR$

8 An asteroid of mass m is approaching the earth, initially at a distance of $10R_e$ with speed v_i. It hits the earth with a speed v_f (R_e and M_e are radius and mass of the earth), then
(a) $v_f^2 = v_i^2 + \frac{2Gm}{M_e R}\left(1 - \frac{1}{10}\right)$ (b) $v_f^2 = v_i^2 + \frac{2GM_e}{R_e}\left(1 + \frac{1}{10}\right)$
(c) $v_f^2 = v_i^2 + \frac{2GM_e}{R_e}\left(1 - \frac{1}{10}\right)$ (d) $v_f^2 = v_i^2 + \frac{2Gm}{R_e}\left(1 - \frac{1}{10}\right)$

9 The earth is assumed to be a sphere of radius R. A platform is arranged at a height R from the surface of the earth. The escape velocity of a body from this platform is fv_e, where v_e is its escape velocity from the surface of the earth. The value of f is
(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$
(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

10 The escape velocity from earth is v_e. A body is projected with velocity $2v_e$. With what constant velocity will it move in the inter planetary space?
(a) v_e (b) $\sqrt{2}\,v_e$
(c) $\sqrt{3}\,v_e$ (d) $\sqrt{5}\,v_e$

11 The radius of a planet is R. A satellite revolves in a circle of radius r with angular speed ω. The acceleration due to gravity on planet's surface would be
(a) $\frac{r^3\omega}{R}$ (b) $\frac{r^2\omega^3}{2R}$
(c) $\frac{r^3\omega^2}{R^2}$ (d) $\frac{r^2\omega^2}{R}$

12 A particle is fired vertically upwards with a speed of 15 kms^{-1} from the surface of earth. The speed with which it moves in intersteller space, is
(take escape speed from surface of earth as 11.2 kms^{-1})
(a) zero (b) 3.8 kms^{-1}
(c) $\approx$ 10 kms^{-1} (d) $\approx$ 4 kms^{-1}

13 The binding energy of a particle (mass = 50 kg) and earth system is ($G = 6.67 \times 10^{-11}$ $\text{Nm}^2\text{kg}^{-2}$; $M_e = 6 \times 10^{24}$ kg; $R_e = 6400$ km)
(a) 3.86×10^{12} J (b) 4×10^6 J
(c) 3.13×10^9 J (d) 1.56×10^9 J

14 A 500 kg satellite is in a circular orbit at an altitude of 500 km above the earth's surface. Because of air friction, the satellite eventually falls to the earth's surface, where it hits the ground with a speed of 2 kms^{-1}. The work done by air friction is ($M_e = 6 \times 10^{24}$ kg, $R_e = 6400$ km)
(a) 1.67×10^{11} J (b) 6×10^9 J
(c) -6×10^9 J (d) -1.67×10^{11} J

15 The earth circles around the sun once a year. The work which would have to be done on the earth to bring it to rest relative to the sun is (Ignore the rotation of earth about its own axis) given that the mass of the earth.
(a) 2.7×10^{34} J (b) 3.7×10^{33} J
(c) -2.7×10^{33} J (d) 6.7×10^{23} J

16 The mass and diameter of a planet are twice those of earth. The period of oscillation of pendulum on this planet will be (if it is a second's pendulum on earth)
(a) $\frac{1}{\sqrt{2}}$ s (b) $2\sqrt{2}$ s
(c) 2 s (d) $\frac{1}{2}$ s

17 The weight of a body on the surface of the earth is 63 N. What is the gravitational force on it due to the earth at a height equal to half the radius of earth?
(a) 34 N (b) 28 N
(c) 39 N (d) 42 N

18 Two bodies of mass m and M are placed a distance d apart. The gravitational potential at the position, where the gravitational field due to them is zero is V. Then,
(a) $V = -\frac{G}{d}(m + M)$ (b) $V = -\frac{Gm}{d}$
(c) $V = -\frac{GM}{d}$ (d) $V = -\frac{G}{d}(\sqrt{m} + \sqrt{M})^2$

19 A particle hanging from a spring stretches it by 1 cm at earth's surface. How much will the same particle stretch the spring at a place 800 km above the earth's surface? (Take, radius of the earth = 6400 km)
(a) 1 cm (b) 2 cm
(c) 0.53 cm (d) 0.79 cm

20 The escape velocity of a body on the Earth's surface is v_e. A body is thrown up with a speed $\sqrt{5}\,v_e$. Assuming that the sun and planets do not influence the motion of the body, the velocity of the body at infinite distance is v_∞. Then, the value of $\frac{v_\infty}{v_e}$ is
(a) zero (b) 1
(c) 2 (d) 3

ANSWERS

SESSION 1	**1** (a)	**2** (b)	**3** (c)	**4** (c)	**5** (c)	**6** (b)	**7** (d)	**8** (a)	**9** (c)	**10** (c)
	11 (d)	**12** (c)	**13** (a)	**14** (b)	**15** (c)	**16** (a)	**17** (d)	**18** (d)	**19** (d)	**20** (a)
	21 (b)	**22** (c)	**23** (c)	**24** (c)	**25** (a)	**26** (b)	**27** (a)	**28** (b)	**29** (a)	**30** (c)
	31 (b)	**32** (a)	**33** (d)	**34** (a)	**35** (c)	**36** (b)	**37** (c)	**38** (a)	**39** (c)	**40** (a)
	41 (b)	**42** (c)	**43** (b)	**44** (c)	**45** (c)	**46** (c)	**47** (b)	**48** (b)	**49** (b)	
SESSION 2	**1** (b)	**2** (a)	**3** (d)	**4** (c)	**5** (a)	**6** (b)	**7** (a)	**8** (c)	**9** (b)	**10** (c)
	11 (c)	**12** (c)	**13** (c)	**14** (d)	**15** (c)	**16** (b)	**17** (b)	**18** (d)	**19** (d)	**20** (c)

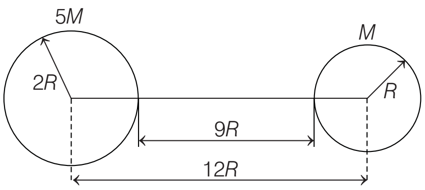
Hints and Explanations

1. $F_{\text{new}} = \dfrac{G \times (2m_1)(2m_2)}{(r/2)^2}$

$= 16\,\dfrac{Gm_1m_2}{r^2} = 16\,F_0$

2 In the space, there is no external gravity. Due to masses of the astronauts, there will be small gravitational attractive force between them. Thus, these astronauts will move towards each other.

3 Suppose, the smaller body cover a distance x before collision, then



$Mx = 5M(9R - x)$ or $x = 45R - 5x$

or $\quad x = \dfrac{45R}{6} = 7.5R$

4 $F = \dfrac{GM_1M_2}{R} = G \times 100 \times 100$

$\dfrac{G}{R} = \dfrac{1}{10000}$

$F' = \dfrac{G}{R} \times 125 \times 75 = \dfrac{G}{R} \times 9375$

$F' = \dfrac{F}{100} \times 9375 \Rightarrow F' = \dfrac{15}{16}F$

5 At height h above earth's surface,

$g' = \dfrac{GM}{(R_e + h)^2} = g \times \left(\dfrac{R_e}{R_e + h}\right)^2$

At depth h below earth's surface,

$g'' = \dfrac{GM\,(R_e - h)}{R_e^3} = g\left[\dfrac{R_e - h}{R_e}\right]$

As, $g' < g''$, then $\Rightarrow F_1 < F_2$

6 At depth $g' = g\left(1 - \dfrac{h}{R}\right)$ or $g\left(1 - \dfrac{d}{R}\right)$

$\dfrac{g'}{g} = \dfrac{1}{n} = \left(1 - \dfrac{d}{R}\right)$

or $\qquad d = R\left(\dfrac{n-1}{n}\right).$

7 Let the original mass of Sun was M_s and gravitational constant G'.

According to the question,

New mass of Sun, $M_s' = \dfrac{M_s}{10}$

New gravitational constant, $G' = 10G$

As, the acceleration due to gravity is given as

$$g = \frac{GM_E}{R^2} \qquad \ldots\text{(i)}$$

where, M_E is the mass of Earth and R is the radius of the Earth.

Now, new acceleration due to gravity,

$$g' = \frac{G'M_E}{R^2} = \frac{10M_EG}{R^2} \qquad \ldots\text{(ii)}$$

$\therefore \quad g' = 10g$ [from Eqs. (i) and (ii)]

This means the acceleration due to gravity has been increased. Hence, force of gravity acting on a body placed on or surface of the Earth increases.

Due to this, rain drops will fall faster, walking on ground would become more difficult.

As, time period of the simple pendulum is

$T = 2\pi\sqrt{\dfrac{l}{g}}$

or $\quad T \propto \dfrac{1}{\sqrt{g}}$

Thus, time period of the pendulum also decreases with the increase in g.

8 The value of g on the surface of the earth, $\quad g \propto \dfrac{1}{R^2}$

At height h from the surface of the earth,

$g' \propto \dfrac{1}{(R + h^2)}$

$\therefore g' = g\,\dfrac{R^2}{(R + h)^2} = \dfrac{9.8 \times (6400)^2}{(6400 + 480)^2}$

$= 8.4 \text{ m/s}^2$

9 Acceleration due to gravity at depth d below the surface of earth,

$$g_d = g\left(1 - \frac{d}{R}\right) \qquad \ldots\text{(i)}$$

Acceleration due to gravity at height h from the surface of the earth,

$$g_h = g\left(1 - \frac{2h}{R}\right) \qquad \ldots\text{(ii)}$$

Here, $\quad g_h = g_d$

$\therefore \quad g\left(1 - \dfrac{2h}{R}\right) = g\left(1 - \dfrac{d}{R}\right) \Rightarrow \dfrac{2h}{R} = \dfrac{d}{R}$

$\therefore \qquad d = 2h$

Thus, $\qquad d = 2 \times 5 = 10\text{ km}$

10 Weight of the body at earth's surface,

$w = mg = 250\text{ N} \qquad \ldots\text{(i)}$

Acceleration due to gravity at depth h from earth's surface

$g' = g\left(1 - \dfrac{h}{R_e}\right)$

Here, $\quad h = R_e/2$

$\therefore g' = g\left(1 - \dfrac{R_e/2}{R_e}\right) = g\left(1 - \dfrac{1}{2}\right) \Rightarrow g' = \dfrac{g}{2}$

$\therefore$ Weight of the body at depth h,

$w' = mg = \dfrac{mg}{2}$

Using Eq. (i), we get $w' = \dfrac{250}{2} = 125\text{ N}$

So, weight of the body will be 125 N.

11 g_h = Acceleration due to gravity at height h above earth's surface

$$= g\left(\frac{R}{R+h}\right)^2 = g\left(1-\frac{2h}{R}\right)$$

g_d = Acceleration at depth d below earth's surface

$$= g\left(1-\frac{d}{R}\right)$$

Given, when $h = 1$ km, $g_d = g_h$

or $\quad g\left(1-\frac{d}{R}\right) = g\left(1-\frac{2h}{R}\right)$

$\Rightarrow \quad d = 2h \quad$ or $\quad d = 2$ km

12 According to the question,

$$\frac{GMm}{(R+h)^2} = \frac{1}{16}\frac{GMm}{R^2}$$

where, m = mass of the body

and $\quad \frac{GM}{R^2}$ = gravitational acceleration

$$\frac{1}{(R+h)^2} = \frac{1}{16R^2}$$

or $\quad \frac{R}{R+h} = \frac{1}{4} \quad$ or $\quad \frac{R+h}{R} = 4$

$$h = 3R$$

13 According to Newton's law of gravitation force,

$$F = \frac{GMm}{R^2}$$

Force on planet of mass M_P and body of mass m is given by

$$F = \frac{GM_P m}{(D_P/2)^2}$$

$$\left[\begin{array}{l}\text{where, } D_P = \text{diameter of planet}\\ \text{and } R_P = \text{radius of planet} = \frac{D_P}{2}\end{array}\right]$$

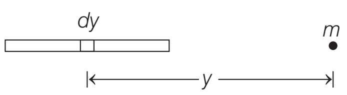
$$F = \frac{4GM_P m}{D_P^2}$$

As we know that, $F = ma$

So, acceleration due to gravity $a = \frac{F}{m}$

$$= \frac{4GM_P}{D_P^2}$$

14 Let a small element dy at a distance y from the mass m be taken, then force due to this element



Gravitational force on the mass due to rod,

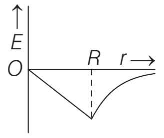
$$F = \int_x^{x+l} \frac{GMdym}{ly^2}$$

$$= \frac{GMm}{l}\left[\frac{1}{x}-\frac{1}{x+l}\right] = \frac{GMm}{x(x+l)}$$

15 Total mechanical energy of any closed system is always negative.

16 $E_{in} = -\frac{GMr}{R^3}$

$E_{out} = -\frac{GM}{r^2}$



17 $\pi R = L$

$\therefore \quad R = \frac{L}{\pi}$,

$$V = -\frac{GM}{R} = -\frac{\pi GM}{L}$$

18 The resulting gravitational potential,

$$V = -2G\left[\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\right]$$

$$\Rightarrow \quad V = -2G\left[1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}\ldots\right]$$

$$\Rightarrow \quad V = -2G\left(1-\frac{1}{2}\right)^{-1}$$

$$\Rightarrow \quad v = -\frac{2G}{\left(1-\frac{1}{2}\right)} = \frac{-2G}{\left(\frac{1}{2}\right)} = -4G$$

19 Gravitational potential at some height h from the surface of the earth is given by

$$V = -\frac{GM}{R+h} \quad \ldots\text{(i)}$$

And acceleration due to gravity at some height h from the earth surface can be given as

$$g' = \frac{GM}{(R+h)^2} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{|V|}{g'} = \frac{GM}{(R+h)} \times \frac{(R+h)^2}{GM}$$

$$\Rightarrow \quad \frac{|V|}{g'} = R+h \quad \text{(iii)}$$

$\because \quad V = -5.4 \times 10^7$ J kg^{-2}

and $\quad g' = 6.0$ ms^{-2}

Radius of earth, $R = 6400$ km.

Substitute these values in Eq. (iii), we get

$$\frac{5.4 \times 10^7}{6.0} = R+h$$

$\Rightarrow \quad 9 \times 10^6 = R+h$

$\Rightarrow h = (9-6.4) \times 10^6 = 2.6 \times 10^6$ m

$\Rightarrow \quad h = 2600$ km

20 Gravitational potential at point $a/2$ distance from centre is given by,

$$V = -\frac{GM}{a} - \frac{GM}{a/2}$$

$$= -\frac{3GM}{a}$$

21 Change in potential energy,

$$\Delta U = -\frac{GMm}{R+2R} - \left(-\frac{GMm}{R}\right)$$

$$= -\frac{GMm}{3R} + \frac{GMm}{R}$$

$$= \frac{2GMm}{3R} = \frac{2}{3}mgR \qquad \left[\because g = \frac{GM}{R^2}\right]$$

22 $\Delta U = \frac{mgh}{\left(1+\frac{h}{R}\right)}$, $h = R$ [given]

$$\therefore \quad \Delta U = \frac{mgR}{2}$$

23 $v = \sqrt{\frac{GM}{r}}$ or $v \propto \frac{1}{\sqrt{r}} \Rightarrow \frac{v_2}{v_1} = \sqrt{\frac{r_1}{r_2}}$

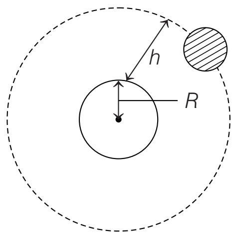
$$\therefore \quad v_2 = \sqrt{\frac{r_1}{r_2}}\, v_1 = \sqrt{\frac{R}{R+\frac{R}{2}}}\, v = \sqrt{\frac{2}{3}}\, v$$

24 Orbital velocity of satellite is $v_0 = \sqrt{gR}$ which is independent of mass of satellite.

Option (c) is correct.

25 Given, height of a satellite

$h = 0.25 \times 10^6$ m



Earth's radius, $R_e = 6.38 \times 10^6$ m

For the satellite revolving around the earth, orbital velocity of the satellite

$$v_0 = \sqrt{\frac{GM_e}{R_e}} = \sqrt{\frac{GM_e}{R_e\left[1+\frac{h}{R_e}\right]}}$$

$$\Rightarrow \quad v_0 = \sqrt{\frac{gR_e}{1+\frac{h}{R_e}}}$$

Substitutes the values of g, R_e and h, we get

$v_0 = \sqrt{60 \times 10^6}$ m/s

$v_0 = 7.76 \times 10^3$ m/s $= 7.76$ km/s

26 Orbital velocity of satellite is given by,

$$v = \sqrt{\frac{GM}{r}}$$

Ratio of orbital velocities of A and B is given by,

$$\Rightarrow \quad \frac{v_A}{v_B} = \sqrt{\frac{r_B}{r_A}}$$

$$= \sqrt{\frac{R}{4R}} = \frac{1}{2}$$

$$\therefore \quad \frac{v_A}{v_B} = \frac{3v}{v_B} = \frac{1}{2}$$

$$\therefore \quad v_B = 6v$$

27 Minimum energy required is given by

$$\frac{mv^2}{2} - \frac{GM_e m}{R_e} = 0$$

$$K_{\min} = \frac{GM_e m}{R_e} = 1.88 \times 10^{11} \text{ J}$$

28 Potential energy of satellite,

$$|U| = \frac{GM_e m}{R_e}$$

Kinetic energy of satellite,

$$K = \frac{1}{2}\frac{GM_e m}{R_e}$$

Thus, $\frac{K}{|U|} = \frac{1}{2}\frac{GM_e m}{R_e} \times \frac{R_e}{GM_e m} = \frac{1}{2}$

29 The only force acting on satellite is gravitational force which is conservative in nature and the work done by conservative force along a closed loop is zero.

30 Potential energy = 2 × (Total energy)

$= 2E_0$

Because we know that, $U = -\frac{GMm}{r}$.

$$E_0 = -\frac{GMm}{2r}$$

31 ∵ Total energy of a satellite at height h is

$$= \text{KE} + \text{PE} = \frac{GMm}{2(R+h)} - \frac{GMm}{(R+h)}$$

$$= \frac{-GMm}{2(R+h)} = \frac{-mg_0R^2}{2(R+h)} \quad \left(\because g_0 = \frac{GM}{R^2}\right)$$

32 $L = mvr$ or $L \propto vr$, $v \propto \frac{1}{\sqrt{r}}$

$\therefore \quad L \propto r^{1/2}$

i.e. with increase in r, L will increase.

33 As we know that, force on satellite is only gravitational force which will always be towards the centre of earth. Thus, the acceleration of S is always directed towards the centre of the earth.

34 From the law of conservation of angular momentum, $L_1 = L_2$

So, $mr_1v_1 = mr_2v_2$

[where, m = mass the of planet
r = radius of orbit
v = velocity of the planet]

$$r_1v_1 = r_2v_2$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \frac{r_2}{r_1}$$

35 $\left(\frac{2GM}{r}\right)^{1/2} \geq C$

36 $v_e = \sqrt{2gR}$, v_e is independent of mass of projectile.

37 $v_o = \sqrt{\frac{GM_e}{R_e}}$

while, $v_e = \sqrt{\frac{2GM_e}{R_e}} \Rightarrow \frac{v_o}{v_e} = \frac{1}{\sqrt{2}} = 1 : \sqrt{2}$.

38 $v_e = \sqrt{2gR} = \sqrt{2\frac{GM}{R^2}R}$ or $v_e \propto \sqrt{\frac{M}{R}}$

Mass is 1000 times and radius is 10 times. Therefore, escape velocity will become 10 times.

$\Rightarrow \quad v_e = 11.2 \times 10 = 112 \text{ kms}^{-1}$

39 $V_e = \sqrt{\frac{2GM}{R}} = C$

$$\Rightarrow R = \frac{2GM}{C^2}$$

$$= \frac{2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(3 \times 10^8)^2}$$

$$= \frac{2 \times 6.67 \times 5.98}{9} \times 10^{-3} \text{m}$$

$$= 8.86 \times 10^{-3} \text{ m} \approx 10^{-2} \text{m}$$

40 Since, the escape velocity of earth can be given as

$$v_e = \sqrt{2gR} = R\sqrt{\frac{8}{3}\pi G\rho}$$

[ρ = density of earth]

$$\Rightarrow \quad v_e = R\sqrt{\frac{8}{3}\pi G\rho} \quad \text{...(i)}$$

As it is given that the radius and mean density of planet are twice as that of earth. So, escape velocity at planet will be

$$v_p = 2R\sqrt{\frac{8}{3}\pi G 2\rho} \quad \text{...(ii)}$$

Divide, Eq. (i) by Eq. (ii), we get

$$\frac{v_e}{v_p} = \frac{R\sqrt{\frac{8}{3}\pi G\rho}}{2R\sqrt{\frac{8}{3}\pi G2\rho}} \Rightarrow \frac{v_e}{v_p} = \frac{1}{2\sqrt{2}}$$

41 From Kepler's law, $T^2 \propto r^3$.

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 \Rightarrow \left(\frac{365 \text{ days}}{T}\right)^2 = \left(\frac{2}{1}\right)^3$$

$\Rightarrow \quad T \approx 129$ days

42 $T^2 \propto r^3, r_1 : r_2 : r_3 = \frac{1}{2} : 1 : \frac{3}{2}$

$$\therefore \quad T_1^2 : T_2^2 : T_3^2 = \frac{1}{8} : 1 : \frac{27}{8} = 1 : 8 : 27$$

43 $\frac{GMm}{r^2} = \frac{mv^2}{r}$ = Centripetal force

$$v^2 \Rightarrow \frac{GM}{r} \quad \text{...(i)}$$

$$T = \frac{2\pi r}{v}$$

$$\Rightarrow \quad T^2 = \frac{4\pi^2r^2}{v^2}$$

On putting the value of v^2 from Eq. (i), we get

$$T^2 = \frac{4\pi^2r^2}{\left(\frac{GM}{r}\right)}$$

$$T^2 = \frac{4\pi^2r^3}{GM} \quad \text{...(ii)}$$

$$T^2 = kr^3 \quad \text{...(iii)}$$

From Eqs. (ii) and (iii), we get

$$\frac{4\pi^2}{GM} = k \Rightarrow GMK = 4\pi^2$$

44 From Kepler's third law

$$T^2 \propto r^3$$

[where, T = time period of satellite
r = radius of elliptical orbit (semi major axis)]

Hence, $T_1^2 \propto r_1^3$ and $T_2^2 \propto r_2^3$

So, $\frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3} = \frac{(3R)^3}{(6R)^3}$ or $\frac{T_2^2}{T_1^2} = \frac{1}{8}$

$$T_2^2 = \frac{1}{8}T_1^2 \Rightarrow T_2 = \frac{24}{2\sqrt{2}} = 6\sqrt{2} \text{ h}$$

45 The line joining the sun to the planet sweeps out equal areas in equal time interval i.e. areal velocity is constant.

$$\frac{dA}{dt} = \text{constant or } \frac{A_1}{t_1} = \frac{A_2}{t_2}$$

where, A_1 = area under SCD

A_2 = area under ABS

$$\Rightarrow \quad t_1 = \frac{A_1}{A_2}t_2$$

Given, $A_1 = 2A_2$

$\therefore \quad t_1 = 2t_2$

46 According to Kepler's, third law

$$T^2 \propto R^3$$

$$\frac{T_2}{T_1} = \frac{(R_2)^{3/2}}{(R_1)} = \frac{(R_1)^{3/2}}{(4R_1)^{3/2}}$$

$$T_2 = \frac{T_1}{8} = \frac{24}{8} = 3 \text{ h}$$

47 $T_1 = T_{\text{earth}} = 1$ yr

$T_2 = T_{\text{neptune}} = 165$ yr

Let R_1 and R_2 be the radii of the circular orbits of the earth and neptune respectively

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} \quad \therefore \quad R_2^3 = \frac{R_1^3T_2^2}{T_1^2}$$

or $$R_2^3 = \frac{R_1^3 \times (165)^2}{1^2}$$

$\therefore R_2^3 = 165^2 R_1^3$ or $R_2 = 30R_1$

48 From conservation of angular momentum, $mv_Ar_A = mv_Pr_P$

$$\Rightarrow \quad v_P = \frac{3.88 \times 10^4 \times 6.99 \times 10^{10}}{4.6 \times 10^{10}}$$

$$= 5.90 \times 10^4 \text{ ms}^{-1}$$

49 Since, moon is satellite of the earth, from Kepler's third law, we have

$$T^2 = \frac{4\pi^2 R^3}{GM_E} \Rightarrow M_E = \frac{4\pi^2 R^3}{GT^2}$$

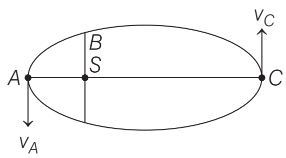
Putting the values, we have

$$M_E = \frac{4 \times 3.14 \times 3.14 \times (3.84)^3 \times 10^{24}}{6.67 \times 10^{-11} \times (27.3 \times 24 \times 60 \times 60)^2}$$

$$M_E = 6.02 \times 10^{24} \text{ kg} \Rightarrow M_E \propto 10^{24} \text{ kg}$$

SESSION 2

1 According to the question,



The figure above shows an ellipse traced by a planet around the Sun, S. The closed point A is known as perihelion (perigee) and the farthest point C is known as aphelion (apogee).
Since, as per the result the Kepler's second law of area, that the planet will move slowly ($v_{\min}$) only when it is farthest from the Sun and more rapidly ($v_{\max}$) when it is nearest to the Sun.
Thus, $v_A = v_{\max}, v_C = v_{\min}$
Therefore, we can write

$$v_A > v_B > v_C \quad \ldots(\text{i})$$

Kinetic energy of the planet at any point is given as,

$$K = \frac{1}{2}mv^2$$

Thus, at A, $K_A = \frac{1}{2}mv_A^2$

At B, $K_B = \frac{1}{2}mv_B^2$

At C, $K_C = \frac{1}{2}mv_C^2$

From Eq. (i), we can write

$$K_A > K_B > K_C$$

2 The acceleration due to gravity on the new planet can be found using the relation

$$g = \frac{GM}{R^2} \quad \ldots(\text{i})$$

But $M = \frac{4}{3}\pi R^3 \rho$, ρ being density.

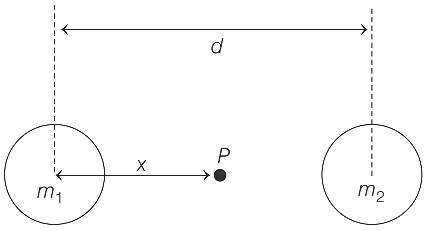
Thus, Eq. (i) becomes

$$\therefore g = \frac{G \times \frac{4}{3}\pi R^3 \rho}{R^2} = G \times \frac{4}{3}\pi R \rho \Rightarrow g \propto R$$

Similarly, $g' \propto R'$

$$\therefore \frac{g'}{g} = \frac{R'}{R} \Rightarrow \frac{g'}{g} = \frac{3R}{R} = 3 \Rightarrow g' = 3g$$

3 Force will be zero at the point of zero intensity



$$x = \frac{\sqrt{m_1}}{\sqrt{m_1} + \sqrt{m_2}}d$$

$$= \frac{\sqrt{81M}\ D}{\sqrt{81M} + \sqrt{M}}$$

$$= \frac{9D}{10}$$

4 By Kepler's law of planetary motion,

$$T^2 \propto r^3$$

$$\therefore \quad T^2 \propto (36000)^3$$

and $(T')^2 \propto (6400 + h)^3$

Therefore, $(T')^2 = T^2\left[\frac{6400 + h}{36000}\right]^3$

$$> T^2\left[\frac{6400}{36000}\right]^3$$

$$> (24)^2\left[\frac{8}{45}\right]^3$$

$$\therefore \quad T' > \frac{24 \times 8 \times \sqrt{8}}{45 \times \sqrt{45}} > 1.8 \text{ h}$$

So, $T' \approx 2$ h

5

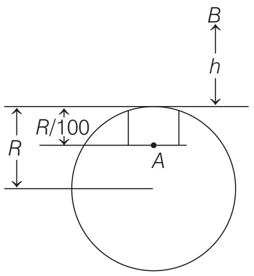
$$W_1 = mg\left(1 + \frac{2h_1}{g}\right),$$

$$W_2 = mg\left(1 + \frac{2h_2}{R}\right)$$

$$\Delta W = W_1 - W_2 = mg\left(\frac{2h_1}{R} - \frac{2h_2}{R}\right)$$

$$= \frac{2mg}{R}(h_1 - h_2)$$

6 Let a particle be projected from A which reaches B at a height h from the surface. It is projected with



$$v_e = \sqrt{\frac{2GM}{R}}$$

KE + PE (at A) = PE (at B)

$$\frac{1}{2}m\left(\frac{2GM}{R}\right) - \frac{GmM}{2R^3}\left[3R^2 - \left(\frac{99}{100}\right)^2 R^2\right] = -\frac{GMm}{(R+h)}$$

$$\Rightarrow \quad R + h = \frac{2R}{0.0199} \Rightarrow h = 99.5\,R$$

7 Potential energy of the object at the surface of the earth $= -\frac{GM_e m}{R}$

PE of the object at a height equal to the radius of the earth $= -\frac{GMm}{2R}$

$\therefore$ Gain in PE of the object

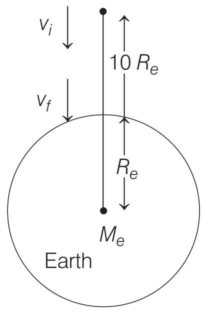
$$= -\frac{GMm}{2R} - \left(-\frac{GMm}{R}\right)$$

$$= +\frac{GMm}{2R} = \frac{gR^2 \times m}{2R} \quad [\because GM = gR^2]$$

$$= \frac{1}{2}mg\,R$$

8 Applying law of conservation of energy for asteroid at a distance $10R_e$ and at earth's surface,

$$K_i + U_i = K_f + U_f \quad \ldots(\text{i})$$



Now, $K_i = \frac{1}{2}mv_i^2$

and $U_i = -\frac{GM_e m}{10R_e}$

$$K_f = \frac{1}{2}mv_f^2$$

and $U_f = -\frac{GM_e m}{R_e}$.

Substituting these values in Eq. (i), we get

$$\frac{1}{2}mv_i^2 - \frac{GM_e m}{10R_e} = \frac{1}{2}mv_f^2 - \frac{GM_e m}{R_e}$$

$$\Rightarrow \frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + \frac{GM_e m}{R_e} - \frac{GM_e m}{10R_e}$$

$$\Rightarrow \quad v_f^2 = v_i^2 + \frac{2GM_e}{R_e} - \frac{2GM_e}{10R_e}$$

$$\therefore \quad v_f^2 = v_i^2 + \frac{2GM_e}{R_e}\left(1 - \frac{1}{10}\right)$$

9 Escape energy = Binding energy of sphere

or $\frac{1}{2} m v'^2_e = \frac{GMm}{R + h}$

or $v'_e = \sqrt{\frac{2GM}{R + h}} = \sqrt{\frac{2GM}{2R}}$ $[\because h = R]$

But at surface of earth,

$$v_e = \sqrt{\frac{2GM}{R}}$$

As given, $v'_e = fv_e$

Hence, $\sqrt{\frac{2GM}{2R}} = f\sqrt{\frac{2GM}{R}}$

$\therefore$ $f = \frac{1}{\sqrt{2}}$

10 $U_i + K_i = U_f + K_f$

$$-\frac{GMm}{R} + \frac{1}{2} m (2v_e)^2 = 0 + \frac{1}{2} mv^2$$

or $-\frac{GM}{R} + 2v_e^2 = \frac{1}{2} v^2$

or $-\frac{2GM}{R} + \frac{8GM}{R} = v^2$

or $v = \sqrt{\frac{6GM}{R}} = \sqrt{3\left(\frac{2GM}{R}\right)}$

$= \sqrt{3(2gR)}$

$= \sqrt{3}\, v_e$

11 $\omega = \frac{v}{r}$,

So, $v = \sqrt{\frac{GM}{r}}$, we get

$\Rightarrow$ $\omega = \frac{1}{r}\sqrt{\frac{GM}{r}} \Rightarrow \omega^2 = \frac{GM}{r^3}$

$\Rightarrow$ $\omega^2 r^3 = g \times R^2 \Rightarrow g = \frac{r^3\omega^2}{R^2}$

12 $\frac{mv^2}{2} - \frac{GMm}{R} = \frac{mv_f^2}{2}$,

where, v is the launching speed.

$\Rightarrow$ $mv^2 - \frac{2\,GMm}{R} = mv_f^2$

$\Rightarrow$ $v^2 - v_e^2 = v_f^2$,

where, $v_e = \sqrt{\frac{2GM}{R}}$ is the escape speed

$v_f = \sqrt{v^2 - v_e^2} \approx 10 \text{ kms}^{-1}$

13 Binding Energy (BE) $= \frac{GM_e m}{R_e}$

$$= \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 50}{6.4 \times 10^6}$$

$= 3.13 \times 10^9$ J

14 Orbital speed of satellite, $v = \sqrt{\frac{GM_e}{r}}$

Initial KE, $K_i = \frac{mv^2}{2} = \frac{GM_e m}{2r}$.

From work-energy theorem,

$$\frac{mv_f^2}{2} - \frac{mv^2}{2} = -\left[-\frac{GM_e m}{R_e} + \frac{GM_e m}{r}\right] + W_{\text{air friction}}$$

$$\Rightarrow W_{\text{air friction}} = \frac{mv_f^2}{2} - \frac{GM_e m}{2r} - \frac{GM_e m}{R_e} + \frac{GM_e m}{r}$$

$$= \frac{500 \times (2 \times 10^3)^2}{2} + 6.67 \times 10^{-11} \times 6 \times 20^{24} \times 500\left[\frac{1}{2 \times 6.9 \times 10^6} - \frac{1}{6.4 \times 10^6}\right]$$

$= -1.67 \times 10^{11}$ J

15 $\omega = \frac{2\pi}{T} = \frac{2\pi}{365 \times 24 \times 3600}$

$= 1.99 \times 10^{-7}$ rad/s

$W = K_f - K_i = 0 - \frac{1}{2} mv^2$ $[\because v = \omega R]$

$= \frac{1}{2} \times 6 \times 10^{24} \times (1.5 \times 10^{11} \times 1.99 \times 10^{-7})^2$

$= -2.7 \times 10^{33}$ J

16 Gravity, $g = \frac{GM}{R^2}$

$\therefore$ $\frac{g_{\text{earth}}}{g_{\text{planet}}} = \frac{M_e}{M_p} \times \frac{R_p^2}{R_e^2} \Rightarrow \frac{g_e}{g_p} = \frac{2}{1}$

Also, $T \propto \frac{1}{\sqrt{g}} \Rightarrow \frac{T_e}{T_p} = \sqrt{\frac{g_p}{g_e}}$

$\Rightarrow$ $\frac{2}{T_p} = \sqrt{\frac{1}{2}}$, $T_p = 2\sqrt{2}$ s

17 Given, $h = \frac{R_e}{2}$

Acceleration due to gravity at altitude h is given by

$$g' = \frac{g}{\left(1 + \frac{h}{R_e}\right)^2} = \frac{g}{\left(1 + \frac{R_e/2}{R_e}\right)^2}$$

$$= \frac{g}{\left(1 + \frac{1}{2}\right)^2} = \frac{g}{(3/2)^2} = \frac{4}{9} g \quad \text{...(i)}$$

Weight of the body at earth's surface,

$w = mg = 63$ N ...(ii)

Weight of the body at altitude $h = R_e/2$,

$w' = mg' = \frac{4}{9} mg$ ...(iii)

Using Eq. (ii), we get

$w' = \frac{4}{9} \times 63 = 28$ N

18 $E_{\text{net}} = 0$

$\therefore \frac{Gm}{x^2} = \frac{GM}{(d - x)^2}$

where, x is distance from m.

$\therefore$ $\frac{x}{d - x} = \frac{\sqrt{m}}{\sqrt{M}}$, $x = \frac{\sqrt{m}}{\sqrt{m} + \sqrt{M}} d$

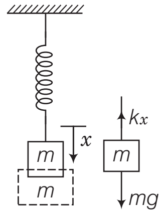
and $d - x = \frac{\sqrt{M}}{\sqrt{m} + \sqrt{M}} d$

$V = -\frac{Gm}{x} - \frac{GM}{d - x}$

$= -\frac{Gm(\sqrt{m} + \sqrt{M})}{\sqrt{m} \cdot d} - \frac{GM(\sqrt{m} + \sqrt{M})}{\sqrt{M} \cdot d}$

$= -\frac{G}{d}(\sqrt{m} + \sqrt{M})^2$

19 Let the mass of the particle be 'm' and the spring constant of the spring is k. The acceleration due to gravity at earth's surface is $g = \frac{GM}{R^2}$.



Let extension of spring be 'x'.

Since, at equilibrium

$kx = mg \Rightarrow x = \frac{mg}{k}$

$\Rightarrow$ $1 \text{ cm} = \frac{GMm}{kR^2}$ ...(i)

At a height 'h' = 800 km, the extension is given by

$x' = \frac{GMm}{k(R + h)^2}$ ...(ii)

From Eqs. (i) and (ii), $\frac{x'}{1\text{cm}} = \frac{R^2}{(R + h)^2}$

$\Rightarrow$ $\frac{(6400 \text{ km})^2}{(7200 \text{ km})^2} = 0.79$

$\therefore$ $x' = 0.79$ cm

20 Applying conservation of mechanical energy,

Energy at initial position = Energy at infinity

$\Rightarrow \frac{1}{2} mv_i^2 - \frac{GmM}{R}$

$= \frac{1}{2} mv_\infty^2$ + GPE at infinity

or

$\frac{1}{2} m(\sqrt{5}\, v_e)^2 - \left(\frac{GM}{R^2}\right) mR = \frac{1}{2} mv_\infty^2 + 0$

or $\frac{1}{2} m(5v_e^2) - mgR = \frac{1}{2} mv_\infty^2$

$\Rightarrow$ $\frac{5v_e^2}{2} - gR = \frac{1}{2} v_\infty^2$

$\Rightarrow$ $\frac{5v_e^2}{2} - \frac{2gR}{2} = \frac{1}{2} v_\infty^2$

$\Rightarrow$ $\frac{5v_e^2}{2} - \frac{v_e^2}{2} = \frac{1}{2} v_\infty^2$ $(\because v_e^2 = 2gR)$

$\Rightarrow$ $\frac{1}{2}(4v_e^2) = \frac{1}{2} v_\infty^2$

$\Rightarrow$ $\left(\frac{v_\infty}{v_e}\right)^2 = 4$

$\Rightarrow$ $\frac{v_\infty}{v_e} = 2$